Chapter 4. Point and non-point solutions of electron equations

1.0. Introduction

1.1. Statement of the problem

The theory of calculation of charge, mass and other characteristics of electron on the basis of the field equations has arisen originally in classical electrodynamics and was developed by W. Kelvin, J. Larmor, H. Lorentz, M. Abraham, A. Poincare, etc. (Pauli, 1958; Ivanenko and Sokolov, 1949). It is based on hypotheses of the field mass and field charge, according to which the particles' own energy or mass is obliged to energy of fields, and the charge of particles is defined by the particles' own fields. These ideas afterwards were transferred to quantum mechanics. But neither classical, nor quantum theories could explain consistently the nature of mass and charge of elementary particles, although for the electron some consecutive theories have been constructed.

1.2. The general requirements to the classical electron mass theory

At first we will address to the hypothesis of electron field mass within the framework of classical electrodynamics (Lorentz, 1916; Ivanenko and Sokolov, 1949).

According to the hypothesis, which has been put forward in the end of the 19^{th} century by J.J. Thomson and advanced by H. Lorentz, M. Abraham, A. Poincare, etc., the electron's own energy (or its mass) is completely caused by the energy of the electromagnetic field of electron. In the same way it is supposed that the electron momentum is obliged to the momentum of the field. Since electron, as any mechanical particle, possesses the momentum and energy, which are together the 4-vector of the generalized momentum, the necessary condition of success of the theory will be the proof that the generalized momentum of an electromagnetic field is a 4-vector.

Thus, for the success of the field mass theory the following conditions should be satisfied at least:

At first, it is necessary to receive final value of the field energy, generated by a particle, which could be precisely equated to final energy of a particle (i.e. product of the mass by the square of the light speed).

At second, the value of a momentum of the field, generated by a particle, must not only be final, but also has the proper correlation with energy, forming with the last a four-dimensional vector.

Thirdly, the theory should manage to deduce the equation of movement of electron.

Fourthly, it is necessary to obtain of electron spin, as a spin of a field (that needs the quantum generalization of the theory of field mass, since a spin is quantum effect).

The analysis shows, that there are two conditions, by which the generalized field momentum G_μ is a 4-vector.

In case of space without charges the size

$$G_\mu = \frac{i}{c}\int T_{\mu 4}(dr), \tag{1.1}$$

will represent a 4-vector if divergence of energy tensor of a field turns into zero:

$$\frac{\partial T_{\mu\lambda}}{\partial x_\lambda} = 0, \tag{1.2}$$

For example, the electromagnetic field, which is located in a space without charges, satisfies similar conditions. In particular, due to this fact, in the photon theory, EM field is characterized not only by energy, but also by momentum.

2) The condition, by which the energy and momentum of an electromagnetic field form a 4-vector at the presence of charges, is formulated by the Laue theorem. According to the last, at the presence of charges the size G_μ is a 4-vector only in the case when in the coordinate system, relatively to which electron is in rest, for all the energy tensor components the following parity is observed

$$\int T^0_{\mu\nu}(d\vec{r}_o) = 0, \tag{1.3}$$

except for the component T^0_{44}, the integral of which is a constant and is equal to full energy of the field, generated by particle (here $(d\vec{r}_o)$ is elementary volume in reference system, in which the electron is in rest). The equality (1.3) expresses a necessary condition, by which the whole particle charge should be in balance.

We can equate this field energy to the particle's own energy, expressing in this way the basic idea of a field hypothesis. According to the last:

$$m_e = \frac{\varepsilon_e}{c^2} = \frac{1}{c^2}\int T^0_{44}(d\vec{r}_o), \tag{1.4}$$

Thus, the mass of a particle from the field point of view can be defined in two ways:

1) proceeding from EM momentum of a field G_1 it is possible to define mass as factor of proportionality between a field momentum and three-dimensional speed of a particle.

2) if we consider the electron's own energy as equal or conterminous to the energy of a field, and mass as the ratio of a field energy $\frac{c}{i}G_4$, to a square of light speed (i.e. as the fourth component of a generalized momentum).

The attempts to execute this program, proceeding from classical linear Maxwell theory, have led to difficulties. In particular, it was not possible to prove the Laue theorem (Tonnelat, 1959). In the classical theory the dynamics (mechanics) and electrodynamics are completely independent from each other. Electromagnetic actions are characterized by component T_0^0 of an energy-momentum tensor of an electromagnetic field. It does not include the energy and momentum of the substance, which should be subsequently inserted. The attempts of Lorentz and Poincare to coordinate the theory on the basis of the assumption that energy of substance has an electromagnetic origin, have not led to a positive result. In Lorentz electron theory (linear in essence) existence of charges it is possible to explain only by introduction of forces of non-electromagnetic origin.

Nevertheless (Sokolov and Ivanenko, 1949), there were also a number of successes, which carried a hope to solve this problem by any change of the theory. The most perspective change of Maxwell-Lorentz theory appeared to be its non-linear generalization.

1.3. Non-linear classical electrodynamics

In the chapter 2 within the framework of CWED we have received the non-linear equation for the electromagnetic (EM) electron and have shown that on sufficient distance from a particle it coincides with the linear Dirac electron equation. But unfortunately the solution of the non-linear equation of the curvilinear electromagnetic wave is not received yet. Its first approach – the non-linear Heisenberg equation - also did not manage to be solved (although here the encouraging results have been received).

We will show the known classical non-linear theories of Gustav Mie, M. Born - L. Infeld, E. Schroedinger etc. represent the approximate solutions of non-linear equation of CWED, which enable us to estimate the sizes of a particle and distribution of a field in approach of spherical electron. Besides, the nonlinear theories find out an opportunity of description of EM electron as point or not point, depending on the used mathematics.

2.0. Gustav Mie approach to the electron theory

2.1. Prior history

Gustav Mie made the first attempt to construct a purely electromagnetic theory of charged particles. (Mie, 1912a; 1912b; 1913; Pauli, 1958; Tonnelat, 1959),). Proceeding from some formally irreproachable hypothetical non-linear generalization of electrodynamics, he managed to construct a theory, which has overcome all difficulties of the classical theory.

As we have said above, in the theory of the electron before G. Mie (Bialynicki-Birula, 1983), the electron was not treated as a purely electromagnetic entity, but it was also made of other stuff, like, for example, Poincare stresses and the mechanical mass. Mie wanted only the electromagnetic field to be responsible for all the properties of the electron. In particular, he wanted the electromagnetic current to be made of electromagnetism.

In order to achieve this goal, Mie assumed that the potential four-vector enters directly into the Lagrangian and not only through the field strength. The generation of the current has been achieved in this manner, but the price was very high. The potentials acquired a physical meaning and the gauge invariance was lost. This property has been found unacceptable by other physicists and the theory of Mie has been shelved for many long years.

2.2. G. Mie theory

In his theory Mie has made two essential steps (Pauli, 1958; Tonnelat, 1959). At first, Mie was the first who suggested in the construction of the theory leaning on a Lagrangian, dependent on fundamental invariants. At second, to get rid of Poincare–Lorentz forces that have non-electromagnetic origin, Mie entered a uniform sight at a field and substance. He set a problem in order to generalize the field equations and the energy-momentum tensor of Maxwell-Lorentz theory in a way that inside the elementary charged particles the repulsion Coulomb forces would counterbalanced by other forces of E origin also, and outside of particles the deviation from ordinary electrodynamics would imperceptible. He assumed that any energy and substance has an electromagnetic origin, and sets as the purpose to deduce properties and characteristics of charges from properties of a field.

About the kind of Lagrangian *L*, which is frequently called a world function, in non-linear electrodynamics it is possible to make some general statements. The independent invariants of an electromagnetic field, which can be formed from bivector $F_{\mu\nu}$ (where $F_{\mu\nu}$ are the tensor components of electromagnetic field strengths) and a vector $A_\mu = \left(i\varphi, \vec{A}\right) = \left(i\varphi, A_i\right) = \left(A_4, A_i\right)$ are the following:

1) The square of bivector $F_{\mu\nu}$: $I_1 = \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$; 2) the square of a pseudo-vector $I_2 = \frac{1}{4}F_{\mu\nu}F^{*\mu\nu}$ (where $F^{*\mu\nu}$ is the dual electromagnetic tensor). 3) The square of a 4-vector of electromagnetic potential A_μ: $I_3 = A_\mu A^\mu$; 4) The square of a vector: $I_4 = F_{\mu\nu}\ A_\mu$; 5) The square of a vector: $I_5 = F^*_{\mu\nu}\ A_\mu$.

Therefore L can depend only on these five invariants. If L is equal to the first of the specified invariants, the field equations are degenerated into ordinary equations of the electron theory for space without charges. Thus, L can noticeably differ from $\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$ only inside the material particles. Invariant 2 can be included in L only as a square, in order not to break the invariance, concerning spatial reflections. Invariants 3-5 break the gauge invariance. Further statements about the world function L cannot be made. Thus, for the selection of L there are an infinite number of opportunities.

Gustav Mie accepted as initial the following Lagrangian:

$$L_{Mi} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - f\left(\pm\sqrt{A_\mu A^\mu}\right), \qquad (2.1')$$

or

$$L_{Mi} = \frac{1}{8\pi}\left(E^2 - H^2\right) - f\left(\pm\sqrt{A_\mu A^\mu}\right), \qquad (2.1'')$$

where f is some function.

Using this Lagrangian (Tonnelat, 1959), Gustav Mie managed to receive the final energy (or mass) of the charged particle as a value completely caused by the energy of the field of this particle. Besides, in this theory the Laue theorem of stability is carried out and the proper correlation between energy and momentum of a particle is reached.

For further analysis it is also useful to mention the attempt of H. Weyl (Pauli, 1958) to interpret on the basis of Mie theory the asymmetry (with respect to distinction of masses) of both sorts of electricity. If L is not a rational function of $\sqrt{A_\mu A^\mu}$, it is possible to put:

$$L^{+}_{Mi} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - f\left(+\sqrt{A_\mu A^\mu}\right), \qquad (2.2')$$

$$L^{-}_{Mi} = \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - f\left(-\sqrt{A_\mu A^\mu}\right), \qquad (2.2'')$$

Thus, if L is a multiple-valued function of the invariants mentioned above, it is obviously possible to choose as world functions for positive and negative charges various unequivocal branches of this function.

2.3. Connection the Mie theory with the CWED

Let's show, that Mie Lagrangian after some additions can be submitted as Lagrangian similar to Lagrangian of CWED (and consequently, of QED).

As we know (Pauli, 1958; Sommerfeld, 1958), the charge density is not invariant concerning Lorentz transformation, but a charge is. Also it is known, that the square of 4-potential, i.e. $I_3 = A_\mu A^\mu$, is invariant concerning Lorentz transformation, but it is not invariant relatively to gauge transformations. But it appears that the product of a square of a charge on I_3 will be an invariant concerning both Lorentz and gauge transformations. Let's show this.

2.3.1. Larmor - Schwarzschild invariant

According to (Pauli, 1958) and (Sommerfeld, 1958), R. Schwarzschild (Schwarzschild, 1903), entered the value

$$S_w = \varphi - \frac{\vec{\upsilon}}{c} \cdot \vec{A}, \qquad (2.3)$$

which he called "electrokinetic potential", and has shown, that this value, being multiplied by density of a charge, forms the relativistic invariant:,

$$L' = \rho S_W = \rho(\varphi - \frac{\vec{\upsilon}}{c} \cdot \vec{A}) = -\frac{1}{c} j_{\mu} \cdot A^{\mu}, \qquad (2.4)$$

where $j_\mu = \{ic\rho, \rho\vec{\upsilon}\}$ is 4-current density, $A^\mu = \{i\varphi, \vec{A}\}$ is 4-potential. Using (2.4) Schwarzschild has formed the following Lagrange function:

$$L = \frac{1}{2}\int \left(H^2 - E^2\right) dV + \int \rho(\varphi - \frac{\vec{\upsilon}}{c} \cdot \vec{A}) dV, \qquad (2.5)$$

and by time-integration (2.5) he has received the function of action.

Thus, in 4-dimensional designations the Schwarzschild Lagrange function density (or Lagrangian) will be written down as follows:

$$\bar{L} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{c} j_\mu A^\mu, \qquad (2.6)$$

and the Lagrange function will be:

$$L = \frac{1}{4}\int F_{\mu\nu} F^{\mu\nu} d\tau - \frac{1}{c}\int j_\mu A^\mu d\tau, \qquad (2.7)$$

(In the note 10 to the book (Pauli, 1958) Pauli marked that before Schwarzschild the same Lagrangian has been suggested by J.J. Larmor (Larmor, 1900)).

Let's consider now the radicand in the Ми Lagrangiane:

$$A_\mu^2 \equiv A_\mu A_\mu = -\varphi^2 + A_i^2, \qquad (2.8)$$

Multiplying it on the squares of density of a charge and a square of a charge, we shall receive accordingly:

$$e^2 A_\mu^2 = -(e\varphi)^2 + (e\vec{A})^2 , \tag{2.9}$$

We will enter the values of density of energy of interaction and energy of electron interaction, accordingly:

$$U_e = \rho\varphi , \ \varepsilon_e = e\varphi , \tag{2.10}$$

and also the density of momentum and the momentum of electron interaction, accordingly:,

$$g_{ei} = \frac{1}{c}\rho A_i , \ p_{ei} = \frac{1}{c} e A_i , \tag{2.11}$$

Then from (2.9) we shall receive:

$$e^2 A_\mu^2 = -\varepsilon_e^2 + \left(c\vec{p}_e\right)^2 , \tag{2.12}$$

Since $\left(\hat{\alpha}_o \varepsilon\right)^2 = \varepsilon^2$, $\left(\hat{\vec{\alpha}}\ \vec{p}\right)^2 = \vec{p}^2$ take place, these expressions can be also written down as:

$$e^2 A_\mu^2 = -\left(\varepsilon_e^2 - c^2 p_{ei}^2\right) = -\left(\left(\hat{\alpha}_0 \varepsilon_e\right)^2 - c^2 \left(\hat{\vec{\alpha}} \vec{p}_{e_i}\right)^2\right), \tag{2.13}$$

Using the above-stated results, for non-linear part of the Mie Lagrangian $L_{Mie}^N = f\left(\pm\sqrt{A_\mu A^\mu}\right)$, we can accept the expression:

$$f\left(\pm\sqrt{A_\mu A^\mu}\right) = e\left(\pm\sqrt{\varphi^2 - c^2 \vec{A}^2}\right), \tag{2.14}$$

Using of Dirac matrixes it is easy to receive the following decomposition:

$$\sqrt{e^2 A_\mu^2} = \mp\left(\hat{\alpha}_0 \varepsilon_e \pm c\hat{\vec{\alpha}}\vec{p}_e\right), \tag{2.15}$$

that gives for non-linear part of Lagrangian the expression:

$$L_{Mie}^{Ne} = \mp\left(\hat{\alpha}_0 \varepsilon_e \pm c\hat{\vec{\alpha}}\vec{p}_e\right), \tag{2.16}$$

Taking into account that

$$\hat{\beta}\ mc^2 = -\left(\varepsilon_p - c\hat{\vec{\alpha}} \cdot \vec{p}_p\right), \tag{2.17}$$

we see that we can enter in the Lagrangian the mass term of the Dirac equation. Thus, it is possible to assert that Mie Lagrangian can be transformed to have the form of the Lagrangian of the non-linear field theory, corresponding to the theory of the electron of CWED and QED also.

The use of these expressions leads to the Dirac equations of electron and positron, and gives the basis to the Weyl‘s attempt to interpret the asymmetry of both sorts of electricity not in connection with mass, but in connection with distinction particle - antiparticle.

Thus, the assumption of Mie that internal properties of electron are described by an electromagnetic field, corresponds to the results of CWED. Actually in the chapter 2 we result that the energy, mass and charge of particle are defined by the inner potentials of this particle. If to accept that potentials inside a particle correspond to an energy-momentum of the particle field, it makes the potentials the physically certain values, which however are not measurable outside of a particle. In other words, the potentials are here the *hidden parameters* of elementary particles.

Do these results contradict to the experimental results of modern physics?

As it is known in classical electrodynamics the potentials play the role of the mathematical auxiliary values and have no physical sense. But as it appears, in framework of quantum mechanics the potentials have physical sense that is proved by Aharonov-Bohm experiment (Aharonov and Bohm, 1959; Feynman, Leighton and Sands, 1989).

As an example of calculation of electron parameters in framework of classical theory, we will consider the results of the Born - Infeld theory (Born and Infeld, 1934) and show, that these results can be considered as some approximation of CWED solution.

3.0. Born-Infeld nonlinear theory

M. Born and L. Infeld revived Mie's theory and proposed a specific model. The Born-Infeld theory (Born and Infeld, 1934) rests on the simplest possible Lagrangian: the square root of the determinant of a second rank covariant tensor. Such a structure automatically guarantees the invariance of the theory under arbitrary coordinate transformations, making the fully relativistic and gauge invariant non-linear electrodynamics.

M. Born and L. Infeld proceeded from the idea of a limited value of the electromagnetic field strength of the electron (which is identical to idea of a limited size of the electron as it is shown below). These reasons led them to the following Lagrangian of the non-linear electrodynamics in the vacuum:

$$L_{BI} = \frac{E_0^2}{4\pi}\left(1-\sqrt{1-\frac{E^2-H^2}{E_0^2}-\frac{\left(\vec{E}\cdot\vec{H}\right)^2}{E_0^4}}\right), \qquad (3.1)$$

where E_0 is the maximum field of electron.

We will consider the most important case of the electrostatic field of the point (spherical symmetric) electron. Putting $\vec{H}=0$, $\vec{E}=-grad\varphi$, $\rho(\vec{x}-\xi)=\delta(\vec{r})\delta(t-s)$, we will find according to (3.1):

$$L_n = \frac{E_0^2}{4\pi}\left(1 - \sqrt{1 - \frac{E_r^2}{E_0^2}}\right) - e\varphi\delta(\vec{r})$$

Then by the help of the variation principle we obtain:

$$-\frac{1}{4\pi}\frac{\partial D_r}{\partial E_r} - \frac{\partial L}{\partial \varphi} = 0$$

where $\vec{D}$ is the electrical induction vector (D-field):

$$D_r = 4\pi\frac{\partial L}{\partial E_r} = \frac{E_r}{\sqrt{1 - \frac{E_r^2}{E_0^2}}}$$

which corresponds to the equation:

$$div\vec{D}_r = 4\pi e\delta(\vec{r})$$

solution of which is:

$$D_r = \frac{e\vec{r}}{r^3}, \tag{3.2}$$

As we see, from point of view of the D-field, the electron should be considered as point particle.

For the electric field (E-field) we obtain:

$$\vec{E}_r = \frac{\vec{D}_r}{\sqrt{1 + \frac{D_r^2}{E_0^2}}} = \frac{e\vec{r}}{r\sqrt{r^4 + r_0^4}}, \tag{3.3}$$

where $r_0 = \sqrt{\frac{e}{E_0}}$ characterize the electron size. In this case, i.e. *from point of view of the electric field (E-field), the electron is not a point particle.* This is very important specificity of the non-linear theory in comparison with the linear theory, which can explain, why experiments on scattering of electron can be interpreted so that the electron looks as a point particle (while the renormalization procedure allows to eliminate the infinities).

From above the electron charge density distribution can be found:

$$\rho = \frac{divE}{4\pi} = \frac{er_0^4}{2\pi\ r(r^4 + r_0^4)^{3/2}}, \tag{3.4}$$

Thus, in respect to the electric field the electron charge can be considered as distributed mainly in volume of radius r_0, since by $r >> r_0$ the density will quickly aspire to zero. Therefore the size r_0 can be considered as effective radius of electron.

Using known values for mass and charge of electron and speed of light, it is possible to receive $r_0 = 2{,}28 \cdot 10^{-13}$ см, which is practically equal to classical radius of electron.

Also it is easy to find value for the maximal field of electron, being a field in the center of the electron (at $r = 0$):

$$E_0 = \frac{e}{r_0^2} = 9{,}18 \cdot 10^{15}\, CGS = 2{,}75 \cdot 10^{20}\, \frac{V}{m} .$$

As it is known, the two types of fields and the two definitions of the charge density, corresponding to them, are also described by the theory of the dielectrics. The value:

$$\varepsilon = \frac{D}{E} = \sqrt{\frac{r^4 + r_0^4}{r^4}} , \qquad (3.5)$$

which is here a function of the position, can be considered as a "dielectric permeability of electron". On large distances from a charge, when $\frac{r_0}{r} \to 0$, ε acquires a value equal to unit as in usual electrodynamics. It is possible to tell that instead of the expression of energy $\frac{e^2}{r^2}$ Born and Infeld take $\frac{e^2}{\varepsilon\, r^2}$, and then the reduction of r is compensated by increase of ε so the full energy remains as final. (It is possible to assume, that the presence of physical vacuum should make the amendment to value of dielectric permeability, and at the same time, to values of potential of electron, its size and other characteristics).

Thus, proceeding from some formal hypothetical non-linear generalization of electrodynamics, it appeared possible (Ivanenko and Sokolov, 1949):

1. to prove the theorem of stability, i.e. to prove, that in the non-linear theory the electron is stable without introduction of forces of non-electromagnetic origin;
2. to receive the final energy (mass) of electron;
3. to receive the final size of its electric charge;
4. to receive the final size of its electromagnetic field.

3.1. Other Lagrangians of nonlinear theories

Also others Lagrangians have been offered for reception of the non-linear theory.

So E. Schroedinger used the following arbitrary combination for Lagrangian:

$$L_{Sch} = \frac{E_0^2}{8\pi} \ln\left(1 + \frac{E^2 - H^2}{E_0^2}\right), \tag{3.6}$$

It was noted (Ivanenko and Sokolov, 1949), that various variants of formal non-linear electrodynamics lead to close values of coefficients, if to take into account, that the electron radius is equal to classical radius of electron. It was also noted, that the basic defect of these theories, as well as of Mie theory, was the arbitrary choice of Lagrangian, which had no connection with the quantum theory, in particular, with Dirac theory, and did not take into account properties of electron, revealed by the last.

We will show that these theories can be considered as approach of the CWED and that they are mathematically connected to the Dirac electron theory.

4.0. The Born-Infeld theory as an approximation of CWED

Since in general case the CWED (see chapter 2), is the non-linear EM theory, therefore its Lagrangian can contain all possible terms with the invariants of electromagnetic theory. Taking into account the gauge invariance the CWED Lagrangian can be written as some function of the following field invariants:

$$\overline{L} = f_L(I_1, I_2), \tag{4.1}$$

where $I_1 = \left(\vec{E}^2 - \vec{H}^2\right)$, $I_2 = \left(\vec{E} \cdot \vec{H}\right)$ are the invariants of electromagnetic field theory.

Apparently, for each problem the function f_L has its special form, which is unknown before. We can suppose that in all cases there is an expansion of the function f_L in the Taylor – MacLaurent power series with some unknown expansion coefficients. It is also obviously that for the most types of the functions $f_L(I_1, I_2)$ the expansion contains approximately the same set of the terms, which are distinguished only by the constant coefficients, any of which can be equal to zero (as an example of such expansion it is possible to point out the expansion of the quantum electrodynamics Lagrangian for particle at the presence of physical vacuum (Akhiezer and Berestetskii, 1965; Weisskopf, 1936; Schwinger, 1951). Generally this expansion looks like:

$$L_M = \frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + L', \tag{4.2}$$

where

$$L' = \alpha \left(\vec{E}^2 - \vec{B}^2\right)^2 + \beta \left(\vec{E}\cdot\vec{B}\right)^2 + \gamma \left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right) + \\ + \xi \left(\vec{E}^2 - \vec{B}^2\right)^3 + \zeta \left(\vec{E}^2 - \vec{B}^2\right)\left(\vec{E}\cdot\vec{B}\right)^2 + \ldots , \qquad (4.3)$$

is a part, which is responsible for the non-linear interaction (here $\alpha, \beta, \gamma, \xi, \zeta, \ldots$ are constants).

The Lagrangian of the Born-Infeld non-linear electrodynamics ican be also expanded into the small parameters $a^2E^2 << 1$ and $a^2B^2 << 1$, where $a^2 = 1/E_0^2$ so that

$$L_{BI} = -\frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + \frac{a^2}{32\pi}\left[\left(\vec{E}^2 - \vec{B}^2\right)^2 + 4\left(\vec{E}\cdot\vec{B}\right)^2\right] + \sum O(\vec{E}^2, \vec{H}^2) , \qquad (4.4)$$

where $\sum O(\vec{E}^2, \vec{H}^2)$ is the series rest with the terms, containing vectors in powers, which are higher than four. Obviously, under conditions $a^2E^2 << 1$ and $a^2B^2 << 1$ on large distance from the center of a particle (where there is a maximal field) the terms of these series really quickly converge, but on small distance from the center it is, apparently, incorrect and here it needs to take into account the terms of higher degrees.

In the chapter 2 we have shown, that at the first approximation Lagrangian of CWED in electromagnetic form can be represented as following:

$$L_N = -\frac{1}{8\pi}\left(\vec{E}^2 - \vec{B}^2\right) + b\left[\left(\vec{E}^2 - \vec{B}^2\right) + 4\left(\left(\vec{E}\cdot\vec{B}\right)^2\right)\right], \qquad (4.5)$$

where b is some constant. Taking into account (4.4), we can write:

$$L_N \approx L_{BI} , \qquad (4.6)$$

and receive in the framework of CWED for EM electron the approach solution, like the solution of Born - Infeld theory, stated briefly above.

For this reasons it can similarly show that the CWED Lagrangian approximately coincides with Lagrangian of Schroedinger and others offered Lagrangians of non-linear theories, allowing us to calculate the corresponding characteristics of electron.

Thus, it is not difficult to answer why "various, from the physical point of view, variants of formal non-linear electrodynamics lead to close values of coefficients": as expansion of non-linear Lagrangian (4.3) shows, all of them are approximately equal among themselves and consequently yield close results.

At the same time, since Lagrangian and equations of CWED completely coincide with Lagrangian and the equations of quantum electrodynamics, the Mie theory and its variant – the Born - Infeld theory, is closely connected with the Dirac theory.